Robust Optimization with Decision Dependent Uncertainty

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INTRODUCTION

- Solutions to decision making problems can be very sensitive to small perturbations in the parameters and such perturbations can make the optimal solutions highly infeasible.
- Robust Optimization (RO) is one of the methods for addressing decision making problems that involves uncertainty in the problem parameters (product demands, travel times, etc).
- In RO, the uncertainty is assumed to belong to a fixed set Ξ. The decision maker seeks to find a solution that performs best in view of the worst case realization of the uncertainty from the set.

$$\Rightarrow \min_{\boldsymbol{x}\in\mathcal{X}} \max_{\boldsymbol{\xi}\in\Xi} (\boldsymbol{C}\boldsymbol{\xi}+\boldsymbol{c})^\top \boldsymbol{x}$$

- In many applications, however, uncertainties are affected by decisions and thus the set is not fixed and may change according to the chosen decision $\boldsymbol{x}$.

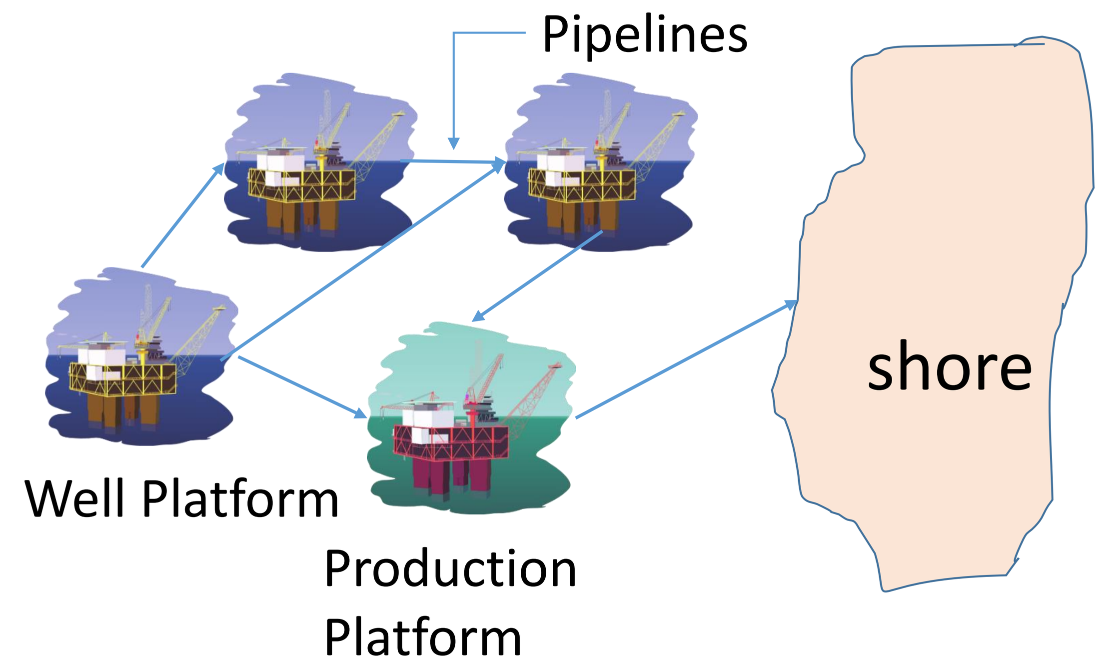
$$\Rightarrow \min_{\boldsymbol{x}\in\mathcal{X}} \max_{\boldsymbol{\xi}\in\Xi(\boldsymbol{x})} (\boldsymbol{C}\boldsymbol{\xi}+\boldsymbol{c})^\top \boldsymbol{x}$$

- We study RO with decision-dependent uncertainty sets (RO-DDU) where the uncertainty set changes depending on the decision.

MOTIVATING EXAMPLES

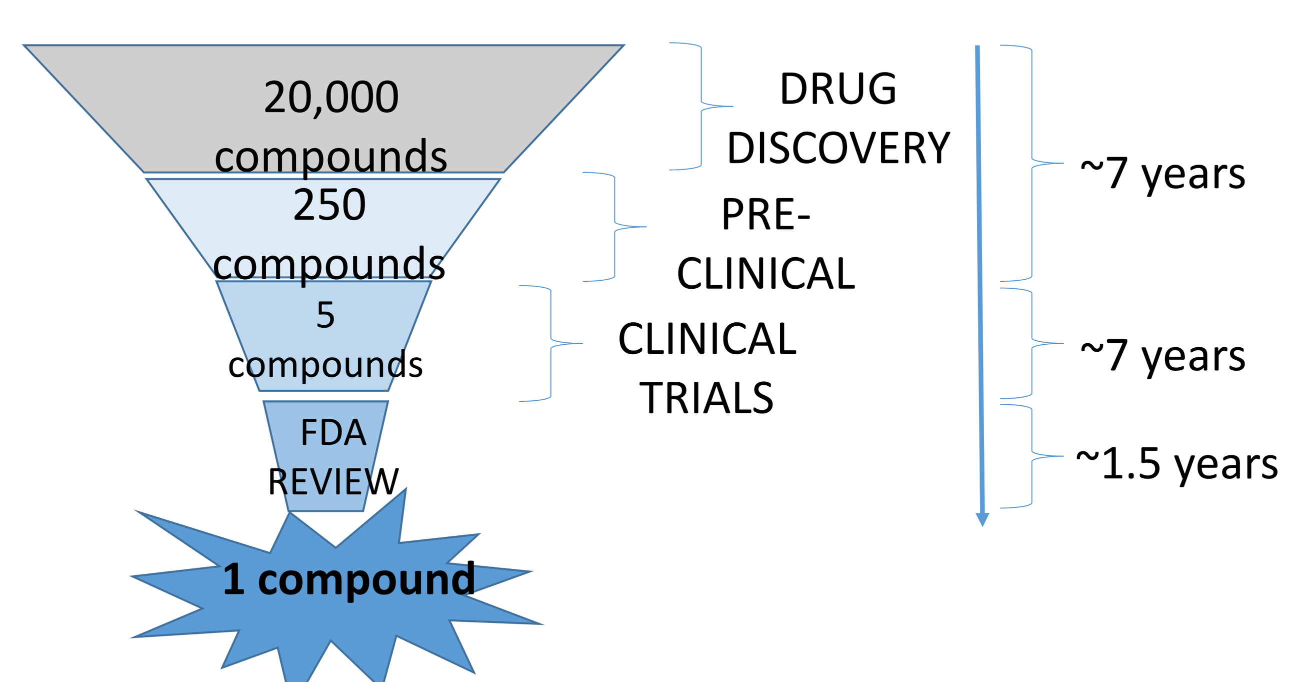
i. Offshore oil & gas exploration

Gas reserves of the fields are uncertain, and the uncertainty is only resolved upon building a well platform on the field. Investment in the fields reduces the uncertainty.



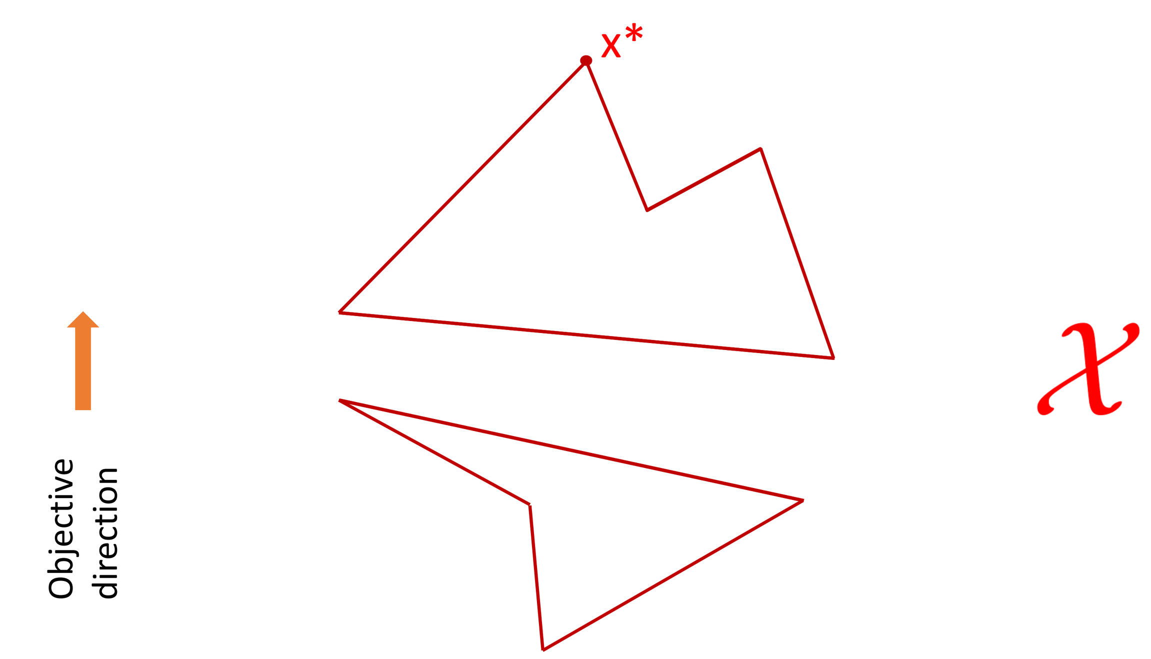
ii. Clinical trials planning

In order for a drug to enter a marketplace, it needs to pass several costly clinical trials. The outcome of each trial will be realized only after the trial is completed. Pharmaceutical companies need to decide on which of the candidate drugs it wants to pursue trials.

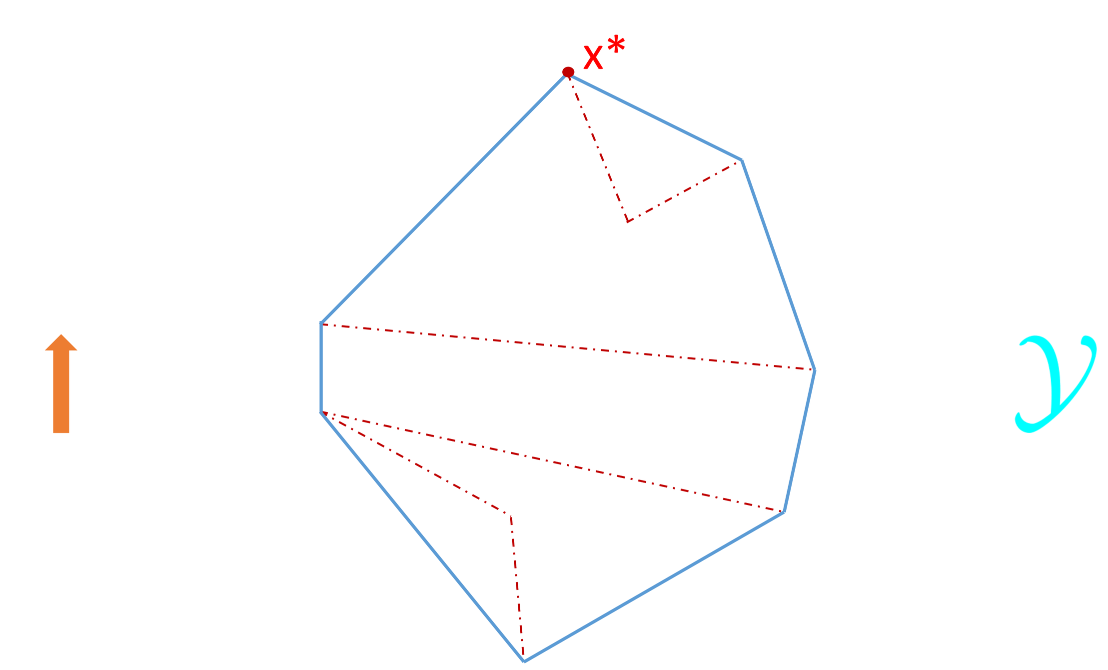


METHODOLOGY (1/2)

- We show that RO-DDU is amenable to a mixed integer non-convex quadratic program with a linear objective function.



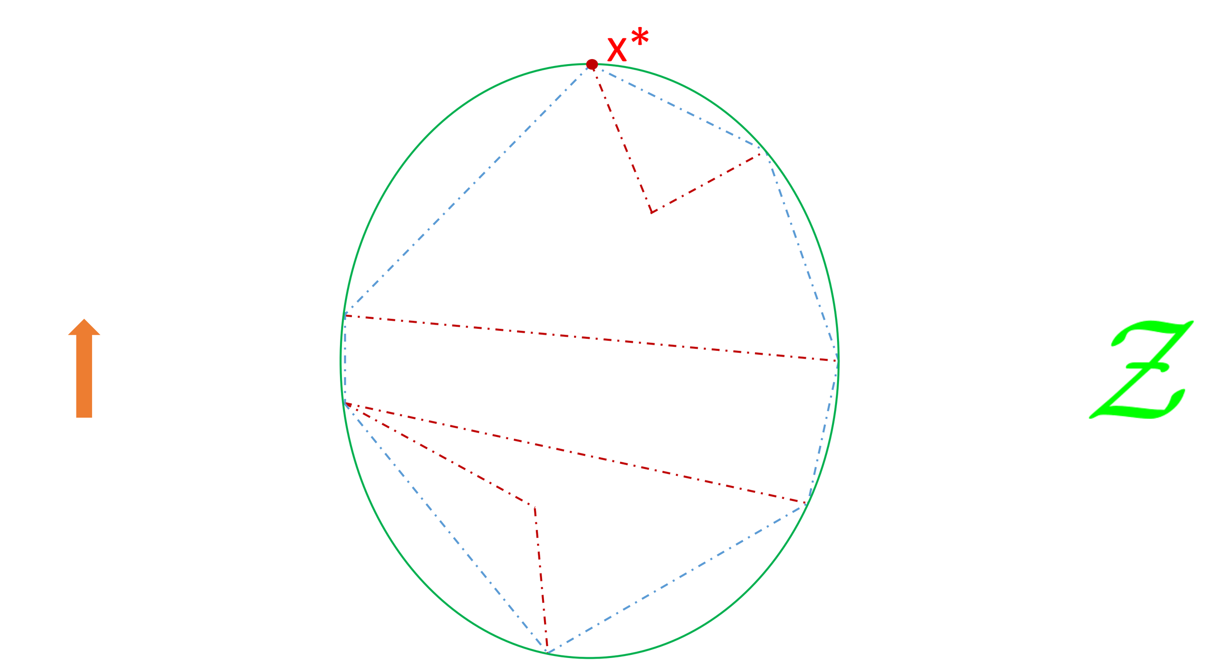
- By adapting the techniques in [1] we can reformulate the resulting non-convex problem exactly as a linear optimization problem over the convex set $\mathcal{Y}$.



- The complexity of the problem is now moved entirely into the set $\mathcal{Y}$. Optimizing over this set is NP-hard, however there exist tractable approximations that can be solved efficiently.

METHODOLOGY (2/2)

- The simplest outer approximation is given by the semidefinite representable set $\mathcal{Z}$ such that $\mathcal{Y} \subseteq \mathcal{Z}$.
- Using the set $\mathcal{Z}$ we obtain a tractable optimization problem which can be solved efficiently using off-the-shelf solvers such as MOSEK, Gurobi, etc.



ONGOING WORK

- Developing inner approximations.
- Cutting plane methods for solving the outer approximation problem.
- Descent algorithms to solve the problem over the set $\mathcal{Y}$ directly.

REFERENCES

[1] Burer S., *On the copositive representation of binary and continuous nonconvex quadratic programs*. Mathematical Programming 120(2009): 479–495.