Johnson Quantile-Parameterized Distributions

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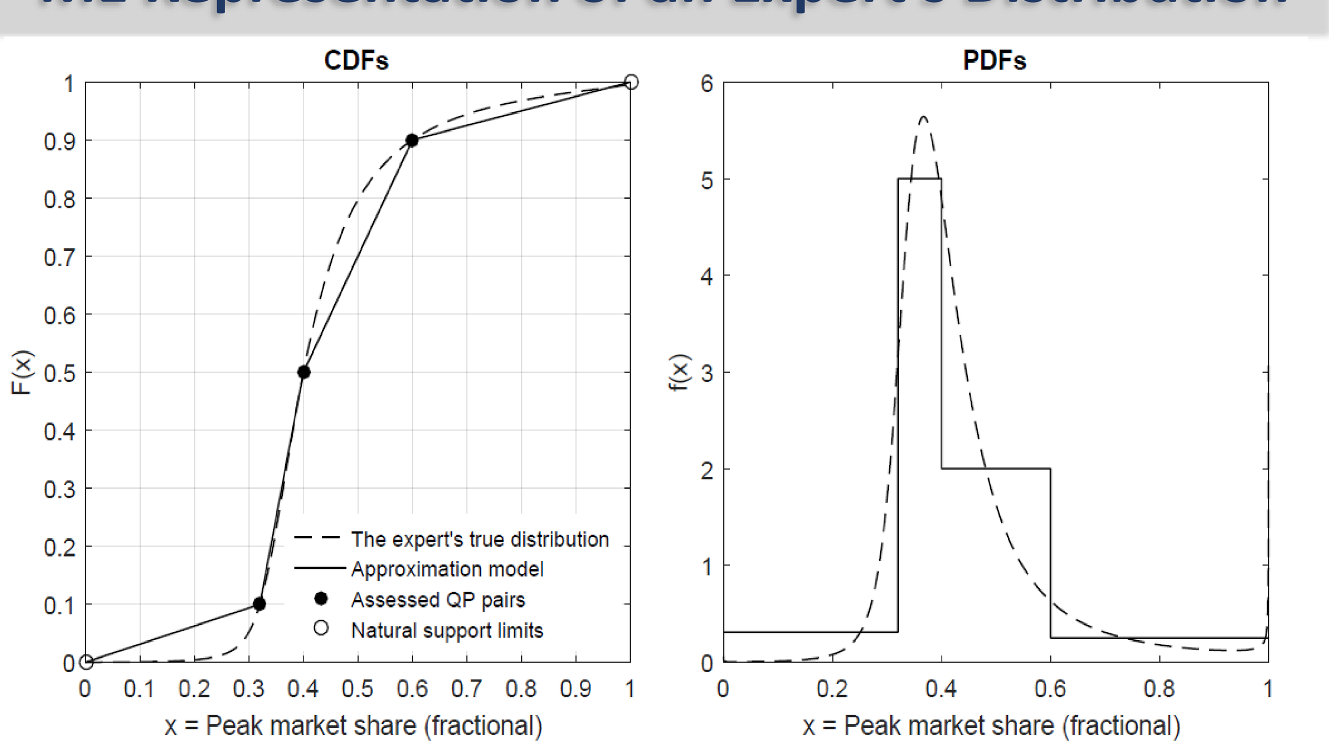
INTRODUCTION (1/2)

In decision analysis, analysts encode uncertainty by eliciting percentile assessments, such as the {10^{th}, 50^{th}, 90^{th}} percentiles, from an expert, and then assigning a distribution to these points in one of several ways.

One approach, **maximum-entropy** (ME), assigns conditional uniform distributions between adjacent quantile assessments. ME is **tractable**, and **honors the QP pairs**, but has several issues:

- It **does not reasonably capture an expert's knowledge** when such knowledge is smooth and continuous over its domain.
- It **cannot capture naturally-occurring shapes**, such as bell-shaped (as with normal, lognormal, etc.).

ME Representation of an Expert's Distribution

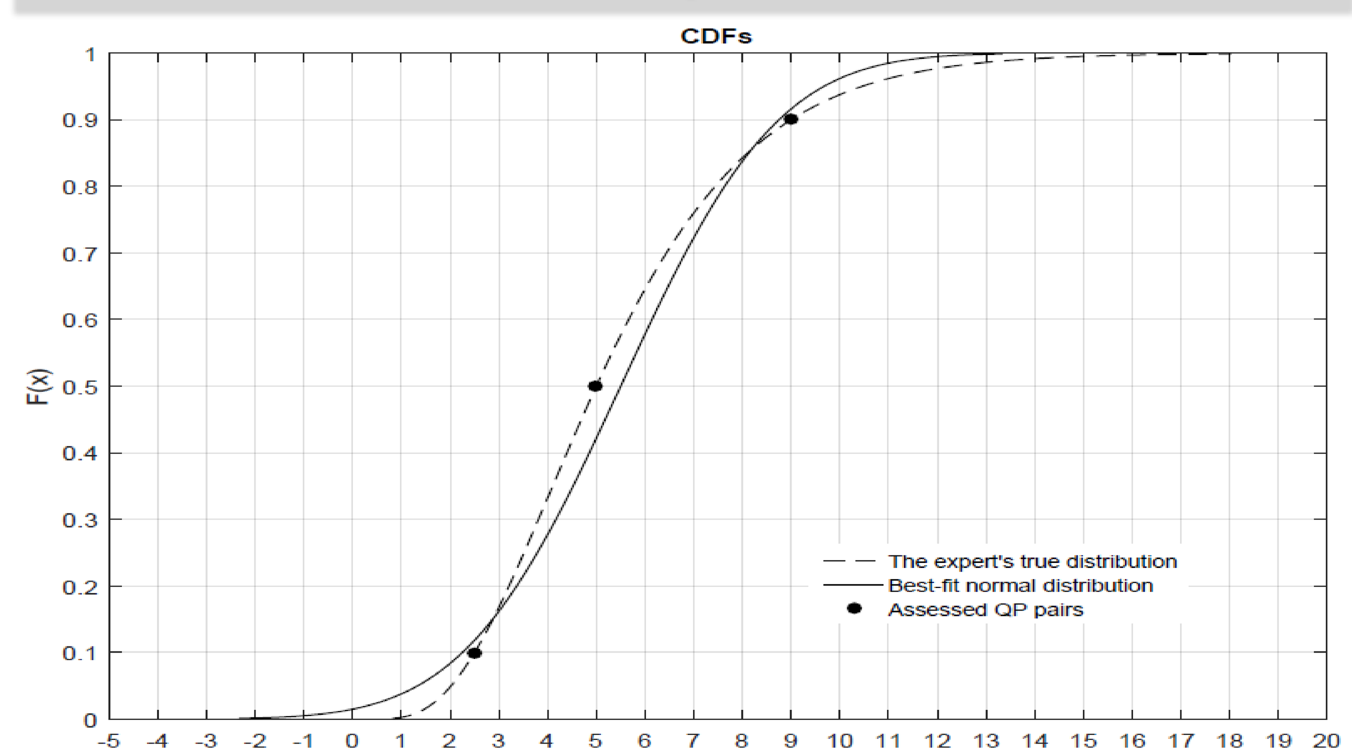


Another approach is **curve fitting** a distribution from a canonical family (normal, lognormal, beta, etc.) to the assessed points (e.g., via least-squares).

INTRODUCTION (2/2)

- Fits may reasonably capture expert knowledge, but **often never honor assessed points**.
- Also, fitting **often involves non-linear, non-convex optimization** over a parameter space.

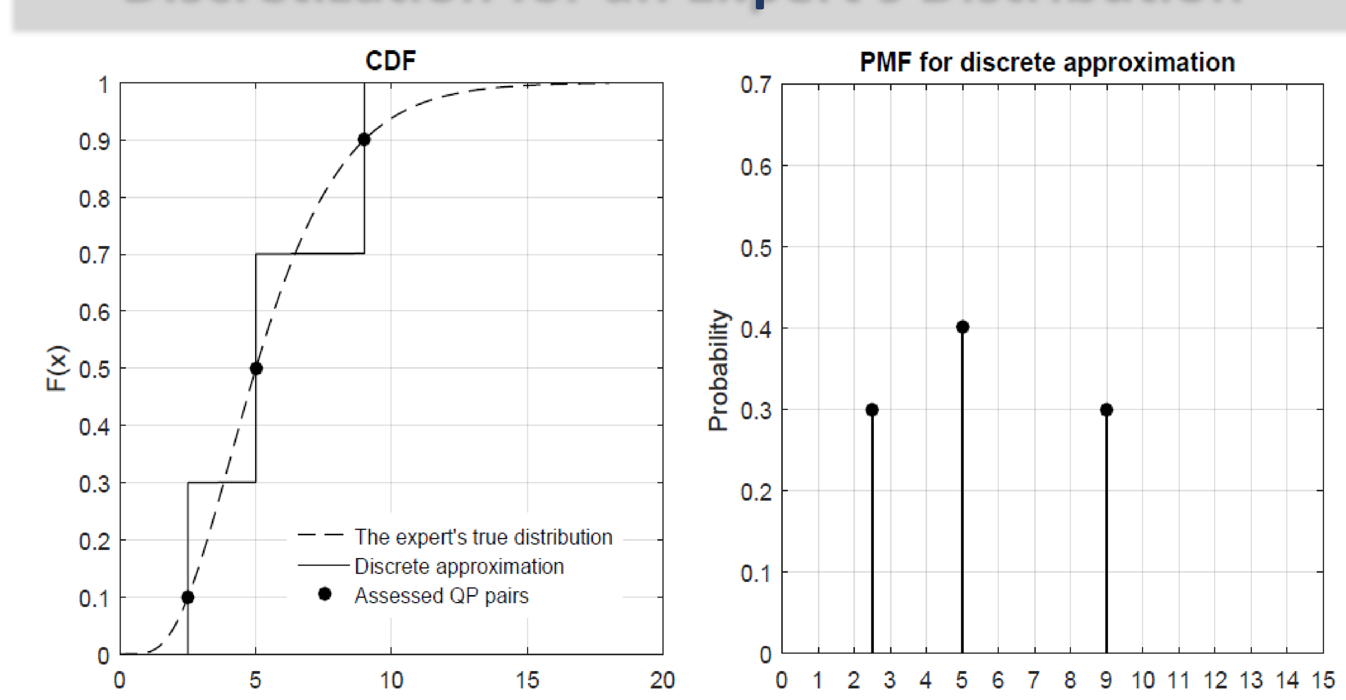
Curve Fit for an Expert's Distribution



Another approach is to apply a discrete distribution to the assessed points. **Discretization honors the assessed points** and is **highly tractable**. However:

- Discretization **poorly captures an expert's knowledge**, particularly by **chopping off distribution tails**.

Discretization for an Expert's Distribution

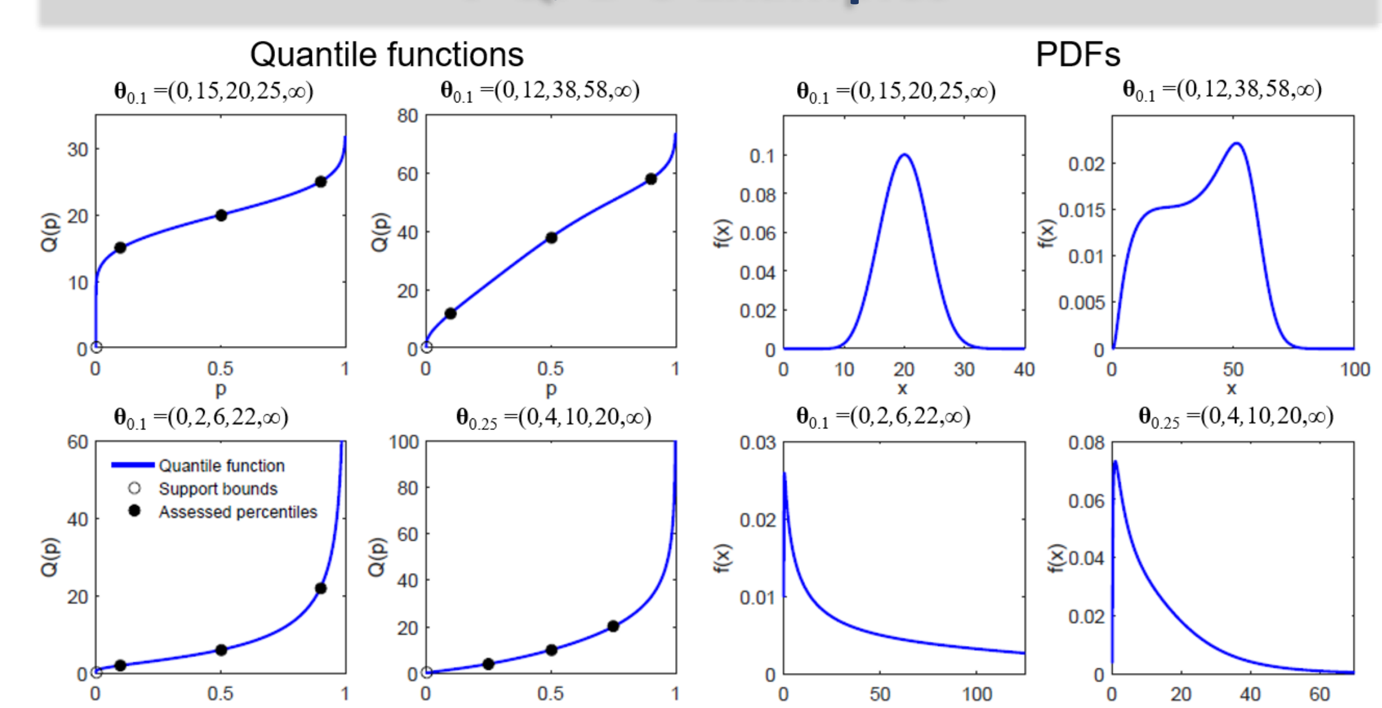


METHODOLOGY

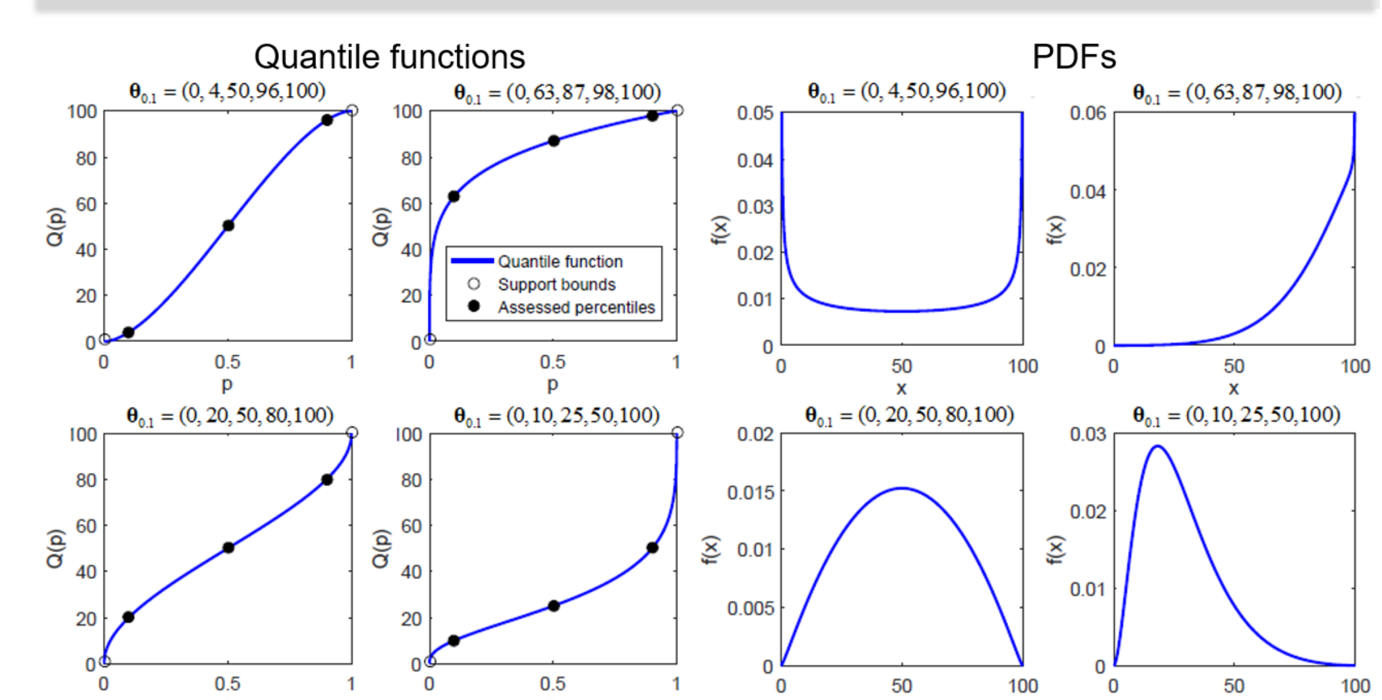
By applying transformations and strategic parameter manipulation to Johnson's SU system, we generate the new "**J-QPD**" distribution system, consisting of the **J-QPD-B** (bounded) and **J-QPD-S** (semi-bounded) subfamilies. J-QPDs are:

- **smooth and continuous.**
- **always honor a symmetric percentile triplet (SPT, e.g., 10^{th}, 50^{th}, 90^{th} percentiles), a specified finite lower bound, and specified upper bound.**
- **Parameterized by the SPT and support bounds in simple form.**

J-QPD-S Examples



J-QPD-B Examples

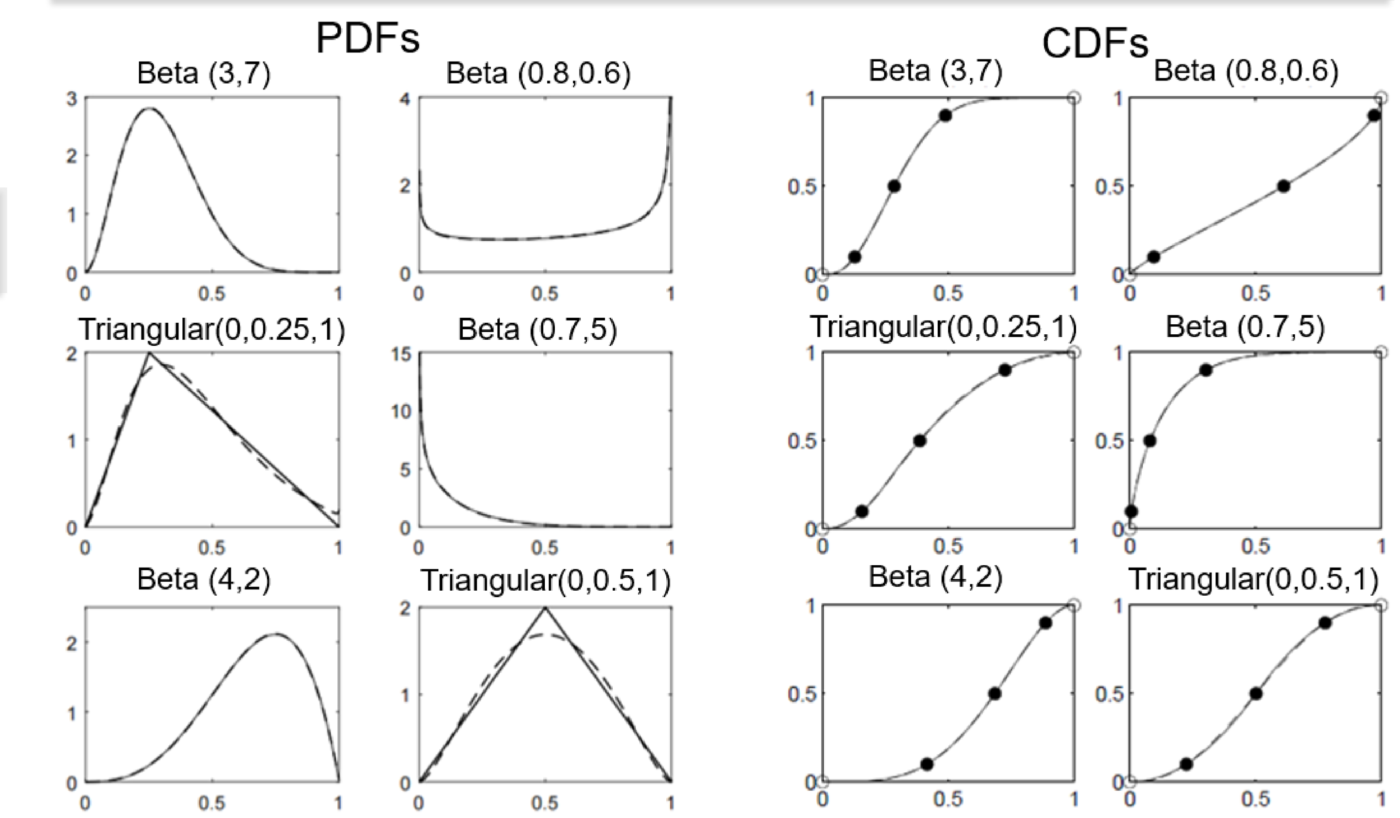


RESULTS

J-QPDs are also **highly flexible**, and can also approximate the shapes of a vast array of commonly-named distributions with **potent accuracy**. For example,

- **J-QPD-B can closely approximate nearly all beta distributions.**
- **J-QPD-S subsumes lognormal, and can closely approximate gamma, Weibull, and many other distributions.**

J-QPD-B Parameterized by {10^{th}, 50^{th}, 90^{th}} Percentiles for Several Beta Distributions



J-QPD-S Parameterized by {10^{th}, 50^{th}, 90^{th}} Percentiles for Several Beta Distributions