Bayesian Identification of Hidden Markov Models with Application to **Condition-Based Monitoring**



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INTRODUCTION

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Project Overview

Program Area: Condition Diagnostic and Prognostic **Objectives:** Develop HMM-based generic condition monitoring methods that are aware of model uncertainty and applicable to partially observable processes.

Deliverables: HMM identification and monitoring code that works with features extracted from sensor signals collected from real manufacturing systems.

Expected Benefits of Project Deliverables

- **Powerful monitoring ability**: based on modeling of unobservable degradation process affected by imperfect maintenance operations. Model uncertainty addressed.
- Wide and easy adaptability: compatible with sensor signal of various types and frequencies requiring minor/moderate configuration for the user.
- **Cost-effective data usage:** use ONLY online process data. No dependency on external information resource.
- **Superior diagnostic performance**: 10% ~ 20% better than PCA to detect known machine abnormality, validated on semiconductor manufacturing process.



METHODOLOGY Hidden Markov Model



Bayesian Estimation for HMM Parameters



Risk Minimization Formulation

 $\widehat{\boldsymbol{\theta}}$:= ar

 $\widehat{\boldsymbol{\theta}} = E(\boldsymbol{\theta}_{0})$

It can be proved the $\boldsymbol{\theta}$ is the solution to the above problem under Dirichlet prior of p_{i} , $\forall i$, when no constraints applied on $\boldsymbol{\phi}$.

distribution $Pr(\boldsymbol{\theta}_0 | \boldsymbol{y}_T)$ so that

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Definition: A hidden Markov model is a doubly embedded stochastic process $\{(X_t, Y_t)\}_{t=0}^{\infty}$ that satisfies the Markov property in X_t and parameterized by

> Pr(Model) Pr(Data|Model) Pr(Data) $= \Pr(Model|Data)$

$$\underset{\substack{(\boldsymbol{\nu},\boldsymbol{P},\boldsymbol{\phi})\\(\boldsymbol{\nu},\boldsymbol{P},\boldsymbol{\phi})}{\text{gmin}} \int_{\Theta} L(\boldsymbol{\theta},\boldsymbol{\theta}_{0}) \operatorname{Pr}(\boldsymbol{\theta}_{0}|\boldsymbol{y}_{T}) d\boldsymbol{\theta}_{0}$$
s.t. $\sum_{i=1}^{m} v_{i} = 1, v_{i} \geq 0, \forall i$

$$\underset{\substack{j=1\\j=1}}{^{m}} p_{ij} = 1, \forall i, p_{ij} \geq 0, \forall i, j,$$
 $\boldsymbol{\phi} \in \Phi.$

Theoretical solution: *posterior mean*

$$\boldsymbol{\theta}_0 | \boldsymbol{y}_T) = \int_{\Omega} \boldsymbol{\theta}_0 \operatorname{Pr}(\boldsymbol{\theta}_0 | \boldsymbol{y}_T) d\boldsymbol{\theta}_0$$

Computational solution: <u>*Monte Carlo approach:*</u> generate samples $\{\boldsymbol{\theta}^{(n)}\}_{n=1}^{N}$ from joint posterior

$$(\boldsymbol{\theta}_0 | \boldsymbol{y}_T) \approx \frac{1}{N} \sum_{n=1}^{N} \boldsymbol{\theta}^{(n)}$$



$$A = \frac{\log \Pr(\mathbf{y}_{t_2} | \boldsymbol{\Theta}) - \log \Pr(\mathbf{y}_{t_2} | \boldsymbol{\Theta})}{t_2 - t_1}$$

$$KS(\Lambda_0, \Lambda_1) = \sup_{x} |F_{\Lambda_0}(x) - KS(\Lambda_0, \Lambda_1)| = \sup_{x} |F_{\Lambda_0}(x)| = KS(\Lambda_0, \Lambda_1) = \sup_{x} |F_{\Lambda_0}(x)| = KS(\Lambda_0, \Lambda_1) = KS(\Lambda_1, \Lambda_1) = KS(\Lambda_1) = KS(\Lambda_1$$



Simulation Configuration: Number of observations in each sequence: 10 to 25; Number of sequences in three experiments: 100/200/300s; Model information: nstate = 4; Gibbs sampling iterations N0 = 100, N = 500. Actual parameters : $\boldsymbol{\nu} = [0.7 \ 0 \ 0.3 \ 1], \boldsymbol{\mu} = [0 \ 5 \ 10 \ 20], \boldsymbol{\sigma}^2 = 100$ [5212], P = [0.90.10.00.0; 00.70.300; 00.950.05; 0001]; Prior distribution: $p_i \sim \text{Dir}(1,1,1,1), \mu_i \sim N(0,10), \sigma_i^2 \sim \text{IG}(1,1) \text{ for } i = 1,2,3,4.$



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that are not easily detectable by sensors. To capture those changes, we are experimenting new modeling and monitoring approaches. Preliminary results are available from application of the new methods to both PECVD and plasma etch processes in semiconductor manufacturing.

2. <u>Analysis of statistical properties of Bayesian</u> estimator

Asymptotic normality of the posterior distribution of HMM parameters is under investigation.